Exercise 1

With the usual current prevailing term structure from the previous exercises, assume that the Libor rates $L(T_2, T_3)$ and $L(T_3, T_4)$ to follow two geometric Brownian motions:

$$dL_t^{2,3} = \mu_{2,3}L_t + \sigma_{2,3}L_t dW_t^1, \quad t \leq T_2$$
$$dL_t^{3,4} = \mu_{3,4}L_t + \sigma_{3,4}L_t dW_t^2, \quad t \leq T_3$$

with $\sigma_{2,3} = 0.25$, $\sigma_{3,4} = 0.3$ and $d\langle W_t^1, W_t^2\rangle = \rho = 0.5$. Consider the following derivative product:

$$max(L(T_2, T_2, T_3) - L(T_3, T_3, T_4), 0),$$

where the cashflow exchange occur at time T_4,

(a) Find the value of such product by simulating the Libor rates in the Libor Market model using the terminal forward measure. You should try to use the the class `ProcessEulerScheme` in the `finmmath` library; alternatively resort to the Euler scheme example in exercise 1.

(b) Experiment with ρ to see how the price reacts to correlation changes and plot the resulting values

Exercise 2

In a Libor market model scenario, for dates $T_0 = 0, T_1, \ldots, T_M$ let the log-normal dynamics of the Libor rate $L_t(t, T_{i-1}, T_i)$ for $i = 0, \ldots M - 1$; be given by:

$$L_t^{T_{i-1},T_i} = \sigma_{T_{i-1},T_i} L_t^{T_{i-1},T_i} dW_t^i \tag{1}$$

for some correlated brownian motions W_t^i, $i = 1, \ldots, m - 1$ such that $d\langle W_t^i, W_t^j\rangle = \rho_{i,j}$ and constants σ_{T_{i-1},T_i} .

(a) Prove that dynamics of $P(t, T_{i-1})/P(t, T_i)$ are of the form

$$d\frac{P(t, T_{i-1})}{P(t, T_i)} = \frac{P(t, T_{i-1})}{P(t, T_i)} \lambda_t^{T_{i-1},T_i} dW_t^i$$

for some suitable volatility process $\lambda_t^{T_{i-1},T_i}$.

(b) Using the Ito lemma find the dynamics of the T_{i-1}, T_i forward bond $FB(t, T_i, T_{i-1}) = P(t, T_i)/P(t, T_{i-1})$

(c) Write down the dynamics of $\lambda_t^{T_{i-1},T_i}$ from exercise 1.