Exercise 1

Assume we are given the zero coupon bond term structure from exercise 2 of Handout 4. Consider a swaption struck at K, entered at year 3 and maturying at time 5, having annual payments.

(a) Prove that pricing a swaption is equivalent to pricing a call option on the swap rate $S_{3,5}$;

(b) Find $S_{3,5}$, and using the Black formula for swaptions, calculate the time 0 value of the swaption assuming $S_t = S_{3,5}$, $t \leq 3$ is log-normally distributed with volatility $\sigma = 20\%$.

Exercise 2

A caplet is said to be paid *in arrears* if the payment of the observed libor rate $L(T_1, T_2)$ is effected at T_1 instead of T_2. Go back to exercise 2 in the handout 3 and using the convexity adjustment repeat the pricing of the caplet by assuming it is paid in arrears. Compute the difference between the valuation in handout 3 and the current: this is the market price of the convexity adjustment.

Exercise 3

Let f be a contingent claim on an asset S_t on which calls and puts are liquidly traded for sufficient strike numbers and densities. Suppose no interest rates are paid.

(a) Prove the *second Breeden and Litzenberger formula:*

$$f(S_t) = f(S_0) + f^{'}(S_0)[S_t - S_0] + \int_{S_0}^{\infty} f^{''}(x)(S_t - x)^+ dx + \int_0^{S_0} f^{''}(x)(x - S_t)^+ dx.$$

(Hint: use exercise 3 from handout 4 together with the integration by parts formula)

(b) Conclude that the value V_f of an European claim f on S expriying at T is apporximately replicated by the following portfolio:

$f(S_0)$	units of cash;
$f^{'}(S_0)$	forward contracts with maturity T on S_t;
$f^{''}(K)\Delta K$	call options with matruity T for all strikes $K \geq S_0$
$f^{''}(K)\Delta K$	put options with matruity T for all strikes $K < S_0$

(c) Let $f(x) = x^3$, $T = 1$ and let S_t be a Black-Scholes asset of volatility $\sigma = 0.3$, $S_0 = 100$; let the strike ranges $K_i = \pm 50\%$ ATM, with a strike gap of 5%. Test point (b) against an Euler scheme simulation of S_t (extract the call and put prices from the Black-Scholes analytic formula in the `finmath` library).