THE INFORMATION PREMIUM FOR NON-STORABLE COMMODITIES

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Abstract. For non-storable commodities forward looking information about market conditions is not necessarily incorporated in today's prices, and the standard assumption that the information filtration is generated by the asset is fundamentally wrong. Electricity and weather are the typical markets we have in mind. We discuss pricing of forward contracts on non-storable commodities based on an enlargement of the information filtration. The method is able to incorporate future information of the spot, which is not accounted for in the present spot price behaviour. The notions of *information drift* and *premium* are introduced, and we argue that significant parts of the supposedly irregular market price of risk observed in electricity markets is in reality due to information miss-specification in the model. Some examples based on Brownian motion and Lévy processes and the theory of initial enlargement of filtrations are considered, where we are able to shed some insight into the nature of the information drift and premium being relevant for the electricity markets. The examples include cases where we take temperature forecasts and CO2 emission costs into account when pricing electricity forwards.

1. Introduction

In this paper we consider the role of forward looking information when pricing forward contracts in markets where the underlying commodity is non-storable. Typical examples of such markets include electricity and weather. In such markets the classical buy-and-hold argument for deriving arbitrage-free forward prices breaks down. Pricing based on the theory of storage and convenience yield have been successfully applied to commodities like oil and gas, but are not relevant for non-storable commodities (see Geman and Vasicek [21] for a discussion of convenience yield in electricity markets, and Geman [19] for an account on the theory of storage). Instead, for non-storable commodites, one usually *defines* the forward price to be the expected risk-neutral price of the commodity at delivery, conditioned on an information filtration (see Benth *et al.* [9]). In mathematical terms, the filtration is usually assumed to be generated by the price process of the underlying commodity. Usually, the filtration is described as the flow of all available market information up to current

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time, which is in line with the mathematical definition as long as the underlying assets are tradeable. However, this is not true in the case of electricity or weather.

Consider the example of a planned maintenance of a major electricity power plant. Let us suppose that the power plant will be taken off the net the whole next month. In a properly working electricity market, this is known information available to all traders, and obviously must affect the prices. However, today's spot price and its information filtration do not necessarily take into account the planned outage. The market knows that for the next month the spot prices will increase by a certain amount since a significant part of the supply side in the market falls out. This again will have a direct impact on the forward contracts with delivery in the maintenance period. However, lack of possibilities to store electricity imply that demand and supply up until today will not reflect the future reduction in production. Thus, the filtration generated by the spot price fails to take these forward looking events into account. Therefore, defining the forward price on the information given by the spot will fail to capture knowledge of future market events.

The objective of this paper is to manifest the crucial role forward looking information is playing for non-storable commodities and to explicitly take future information into account when pricing electricity forwards. To achieve this, we propose to enlarge the information filtration used in the derivation of forward prices. This approach is based on the theory for "enlargment of filtrations", see e.g. Protter [31]. In this respect, we define the notion of an *information premium* as the difference between forward prices based on the enlarged filtration and the spot price information (see (2.7)). The information premium can be both positive or negative, effectively measuring the premium charged by either consumers or producers as a function of available market information. Closely related to the the information premium is the *information drift*. It is possible to find a process which becomes a Brownian motion under the enlarged filtration. This process takes the form of a drifted Brownian motion with respect to the filtration of the spot. By changing the driving noise, the effect will be an additional drift term in the spot dynamics. This information drift is the ananlogue to the "market price of risk" resulting from a change of probability measure using a Girsanov or Esscher transform. Note that in a recent paper by Cartea, Figueroa and Geman [13], the spot price dynamics of UK electricity is assumed to depend explicitly on forward looking capacity information. The dependence comes in the drift of the price dynamics. Although our analysis holds for non-storable commodities in general, we shall mainly be concerned with electricity as the prime example.

Another motivation for studying the information premium is the interplay with the risk premium. In traditional no-arbitrage pricing theory, the risk-neutral probability introduces a so-called market price of risk, as mentioned above. The market price of risk is closely linked to the risk premium, and in applications it is usually not explained but simply specified parametrically and estimated from market data. Following this approach, several studies (see e.g. [12], [15]) conclude that the market price of risk and/or the risk premium in electricity markets behave rather irregular: they attain both positive and negative values, vary with time to maturity of the forward contracts, and are seemingly random in nature. We argue that at least parts of this behaviour may be attributed to information miss-specification in the model. When one does not include forward looking information in the pricing model, the calibrated market price of risk has to capture the possibly very irregular information premium.

Using the enlargement-of-filtration approach, we derive forward prices for specific arithmetic two-factor spot models, where the innovations are allowed to be Lévy processes. We analyse a situation where the electricity spot prices are correlated with temperatures and the market has available temperature forecasts. The information premium and drift are calculated, providing an explanation for the difference between the forward and predicted spot prices observed in the market. In particular, in the Nordic electricity market Nord Pool, where demand is largely temperature driven, the information premium in terms of the temperature forecasts is positive with decreasing temperatures. This is in line with the expected behaviour of the forward prices since a large part of the electricity production goes to household heating in the Nordic region. The main result for this example is the explicit expression for the information drift in terms of temperature forecasts.

Another focus point of our study is the pricing of CO2 emission rights and their effect on electricity prices. For example, in the German electricity market EEX, a significant part of the power production is coal and gas fired. At EEX one could in the autumn of 2007 observe a sharp increase between forward prices delivering in December 2007 and January 2008. This increase can largely be explained by the costs of emitting CO2, which were not effectively incurred on the market before January 2008. This was known, of course, by the market participants, and explicitly accounted for in the forward prices. However, the spot prices in the autumn of 2007 were not influenced by the CO2 emission costs to be charged in January the following year. We discuss the situation in detail, and provide a modelling framework capturing this situation.

The rest of this paper is organized as follows: in the next section we discuss further our pricing approach, and define the forward price based on an enlarged filtration. Section 3 moves on with a more detailed mathematical analysis, including some relevant examples where we can explain the risk premium in terms of information. Moreover, we also provide a foundation for understanding why the sign of the risk premium in the electricity market may change. Finally, we conclude in the last section and give some directions for future research.

2. Forward pricing and information for non-storable commodities

In this section we motivate and introduce our proposed pricing framework for forward contracts in non-storable commodity market. Our main focus is to argue for the relevance of forward looking market information which will lead us to the notions of *information premium* and *drift*.

2.1. **The pricing framework.**

Let us start by recalling some standard theory of forward pricing as a foundation for our further discussion. Consider a spot with dynamics given by a semimartingale process $S(t)$, defined on a complete probability space $(\Omega, \mathcal{F}, P)$ equipped with a filtration $\mathcal{G}_t$ satisfying the usual conditions. The filtration $\mathcal{G}_t$ models the flow of available information. If the spot can be traded without frictions, a forward contract delivering at time T can be hedged perfectly by the buy-and-hold strategy, and in the absence of arbitrage, the price is expressible as (see eg Duffie [17])

$$f(t,T) = \mathrm{e}^{r(T-t)}S(t)\,. \tag{2.1}$$

Here, the risk-free interest rate is $r > 0$. Furthermore, the price can be expressed in terms of a conditional expectation under a risk-neutral probability (equivalent martingale measure) Q by,

$$f(t,T) = \mathbb{E}_Q\left[S(T)\,|\,\mathcal{G}_t\right]\,. \tag{2.2}$$

The basic ingredients here are the filtration $\mathcal{G}_t$, to which $S(t)$ is adapted, and Q, which turns the discounted spot price into a martingale (possibly local). In order to perform the buy-and-hold strategy, we must *store* the spot without costs, which is not feasible for a non-storable commodity. Taking the electricity market as an example, power can be stored indirectly by hydro producers using water reservoirs, however, this is extremely inefficient. Weather is obvioulsy not storable.

The theory of forward pricing in commodity markets extends the arbitrage-free relation (2.2) to include for storage and transportation costs, highly relevant for commodities like oil and metals (see eg Hull [23]). The underlying spot products (like oil, say), can be purchased and stored, and thus the buy-and-hold strategy is implementable, however at a cost of storage and transportation at delivery. In these markets, one also argues for the so-called *convenience yield*, which assigns a certain positive yield to holding the underlying commodity rather than being long a forward contract on the same asset (see Hull [23] for more details and references). All in all, the aspects of convenience yield and storage costs implies a modification of the spot-forward relation (2.1) based on arbitrage arguments as follows. For a constant yield δ, the forward price can be represented as

$$f(t,T) = \mathrm{e}^{(r-\delta)(T-t)}S(t)\,. \tag{2.3}$$

The convenience yield is positive, while the storage and transportation costs incur a negative contribution to δ. The relation (2.3) may be expressed in terms of a conditional expectation as well, by choosing appropriately a risk-neutral measure Q. There exists many extensions of the theory of convenience yield and storage, however, the basic underlying condition is the *storability* of the commodity. Hence, in the case of electricity markets, say, this approach is inappropriate. The non-storability feature of the commodity also rules out the existence of a convenience yield (see Eydeland and Wolyniec [18] and Geman [19]).

For a non-storable commodity, the exact relation between the spot and forward is not clear. The usual approach is to resort to the general principle from no-arbitrage pricing theory and to *define* the forward price in terms of the spot as follows:

Definition 2.1. *The forward price $f_{\mathcal{G}}^Q(t,T)$ at time t of a contract with delivery at time T is defined as*

$$f_{\mathcal{G}}^Q(t,T) \triangleq \mathbb{E}_Q\left[S(T)\,|\,\mathcal{G}_t\right] \tag{2.4}$$

where the filtration $\mathcal{G}_t$ models the flow of the available information in the market. Furthermore, Q is a risk-neutral probability.[1]

There are a few things worth commenting on this way of defining the forward price. Due to non-storability, we can not use the spot as part of a portfolio strategy. In this sense the spot is not tradeable, and therefore it does not need to be a (local) martingale under the risk-neutral probability. Hence, the risk-neutral probability Q can be any (equivalent) probability, and we generally have a wide range of candidate probabilities to choose from. It is then by far not clear which is the "correct" one. One usually selects Q from a parametric class, essentially changing the drift of S in the most frequently encountered models. Hence, the resulting risk-neutral forward dynamics has a parametric term coming from Q which next can be estimated using historical forward price data. The drift is commonly referred to as the *market price of risk* since it is a crucial ingredient in measuring the deviation from the predicted spot at delivery

$$f_{\mathcal{G}}(t,T) = \mathbb{E}\left[S(T)\,|\,\mathcal{G}_t\right] . \tag{2.5}$$

Definition 2.2. *The risk premium is defined as the difference between the forward price $f^Q_{\mathcal{G}}(t,T)$ and the predicted spot $f_{\mathcal{G}}(t,T)$, denoted by*

$$R^Q_{\mathcal{G}}(t,T) \triangleq f^Q_{\mathcal{G}}(t,T) - f_{\mathcal{G}}(t,T) = \mathbb{E}_Q\left[S(T)\,|\,\mathcal{G}_t\right] - \mathbb{E}\left[S(T)\,|\,\mathcal{G}_t\right] . \tag{2.6}$$

The market is in normal backwardation when the premium is negative, meaning that the forward price is lower than the predicted spot price. In backwardation, one interprets the risk premium as the additional fee speculators charge in order to take on the risk from producers, who, on the other hand, hedge their production. We refer to, e.g. Benth, Cartea and Kiesel [3] for a discussion of these issues in the context of energy markets.

Although the pricing rule in Definition 2.1 gives a market consistent way to derive a forward price dynamics, it seems to be difficult to apply in practice. The connection between spot and forward is not clear, and thus it is hard to find any reasonable measure Q flexible enough to model the electricity forward in a sound way, and at the same time being feasible for analysis. In addition, the approach does not provide any satisfactory financial explanation for the formation of forward prices based on the spot. Several authors have studied the market price of risk and the implied risk premium in electricity markets and found a rather irregular and random behavior. Geman and Vasicek [21] find that the risk premium for forward contracts with short maturities is positive for data from the Pennsylvania-New Jersey-Maryland (PJM) market. This can be explained by the consumer's desire to ensure short-term delivery of electricity, and because of the big volatility, a positive premium is accepted. For longer-maturing contracts, the picture may change due to utilities that wants to hedge their long-term production. Later studies by Longstaff and Wang [27] and Diko, Lawford and Limpens [15] have confirmed these results, and extended them to other markets and for longer maturities. Further references that confirm the irregular behavior of risk premia include Cartea and Figueroa [12] and Lucia and Torro [29]. In this article we

[1]Electricity forwards deliver over a period rather than at a fixed future time. In this paper we ignore this fact to keep matters simpler.

want to argue that significant parts of this behavior of the risk premium, or market price of risk, are due to information miss-specification. We propose to use the flow of information about future market conditions available to the participants as the key to understand how forward prices are formed.

The information flow $\mathcal{G}_t$ in Definition 2.1 should represent all available information to the market at time t, where forward looking events that the market has knowledge of are included (see e.g. Eydeland and Wolyineck [18]). As mentioned in the introduction, the usual assumption for traded assets in financial markets is that all information is incorporated in asset price behavior and that $\mathcal{G}_t = \mathcal{F}_t$, where $\mathcal{F}_t$ is defined as the filtration generated by the asset or its noise drivers. To the best of our knowledge, at least in concrete applications and calculations, this assumption is transferred to electricity markets without further considerations. However, as argued before, this presumption is fundamentally wrong for non-storable assets. We propose to explicitly enlarge the information flow available at time t in the market, to account for forward looking information. This leads us to the introduction of the information premium.

Definition 2.3. *The information premium is defined as*

$$I_{\mathcal{G}}(t,T) \triangleq f_{\mathcal{G}}(t,T) - f_{\mathcal{F}}(t,T) = \mathbb{E}\left[S(T) \,|\, \mathcal{G}_t\right] - \mathbb{E}\left[S(T) \,|\, \mathcal{F}_t\right] . \tag{2.7}$$

Here $\mathcal{F}_t$ is the filtration generated by the spot price, and $\mathcal{G}_t$ is the filtration representing all available information including forward looking events.

The information premium measures the added value in forward prices implied by the supplemental information contained in $\mathcal{G}_t$ compared to $\mathcal{F}_t$. Let us explain this in more detail (assuming that $S(T)$ has finite variance): since $\mathcal{F}_t \subset \mathcal{G}_t$, the use of iterated conditioning implies

$$I_{\mathcal{G}}(t,T) = \mathbb{E}\left[S(T) \,|\, \mathcal{G}_t\right] - \mathbb{E}\left[\mathbb{E}\left[S(T) \,|\, \mathcal{G}_t\right] \,|\, \mathcal{F}_t\right] .$$

Hence, the information premium is the residual random variable from projecting $\mathbb{E}\left[S(T) \,|\, \mathcal{G}_t\right]$ onto the space $L^2(\mathcal{F}_t, P)$. In this sense the information premium measures how much more information is contained in $\mathcal{G}_t$ than $\mathcal{F}_t$. As an immediate consequence, due to the orthogonality,

$$\mathbb{E}\left[I_{\mathcal{G}}(t,T) \,|\, \mathcal{F}_t\right] = 0 , \tag{2.8}$$

for all $t \leq T$. Further, note that we get the decomposition

$$\begin{aligned} R^Q_{\mathcal{F}}(t,T) &\triangleq f^Q_{\mathcal{F}}(t,T) - f_{\mathcal{F}}(t,T) \\ &= f^Q_{\mathcal{G}}(t,T) - f_{\mathcal{G}}(t,T) - f^Q_{\mathcal{G}}(t,T) + f^Q_{\mathcal{F}}(t,T) + f_{\mathcal{G}}(t,T) - f_{\mathcal{F}}(t,T) \\ &= R^Q_{\mathcal{G}}(t,T) - I^Q_{\mathcal{G}}(t,T) + I_{\mathcal{G}}(t,T) . \end{aligned}$$

Thus, if the risk premium $R^Q_{\mathcal{F}}(t,T)$ is measured based on price information only, the effects of the information premium (under Q and P) in addition to the true risk premium $R^Q_{\mathcal{G}}(t,T)$ are also captured due to the information miss-specification in the model. Also remark that the forward price $f_{\mathcal{G}}(t,T)$ defined according Def. 2.1 is a martingale process with respect to the filtration $\mathcal{G}_t$, but not necessarily with respect to $\mathcal{F}_t$.

In this paper, our main focus is on analyzing the structure and effects of the information premium where we concentrate on its contribution $I_{\mathcal{G}}(t,T)$ under P (the study of $I_{\mathcal{G}}^{Q}(t,T)$ can be done in an analogue way). The general mathematical framework that provides the tools for our analysis is the theory of enlargement of filtrations which has been developped in [25], [24], [26]. An application of this theory considered by several authors is to study insider trading in common stock markets (see [1], [10], [16] and references therein to mention only a few). We here argue for the use of this theory in pricing of forwards on non-storable assets.

From a general perspective, one can define rather rich structures to model the information flow. However, from a more concrete viewpoint, the case of Brownian motion and more general Lévy processes with certain types of information additions lead to explicit results where the effects of adding information may be understood in more detail. Hence, for the analysis in this paper we resort to models of the spot driven by Lévy processes, and to certain kinds of knowledge of the future spot.

In the same spirit as the risk premium is associated to an additional drift given by the market price of risk, the information premium is associated to an additional drift. We have the following definition

Definition 2.4. *Let L be a Lévy process with respect to the filtration $\mathcal{F}_t$ and $\mathcal{F}_t \subset \mathcal{G}_t$. Assume there exists a $\mathcal{G}_t$-adapted process $\theta(t)$ such that*

$$U(t) - \int_0^t \theta(s)\,ds$$

is a $\mathcal{G}_t$-martingale. Then we call $\theta(t)$ for the information drift.

Such a drift θ comes in as a "yield" implied to the forward price, which reflects the additional information available. The extra information added to the spot-generated filtration leads to essentially the same result as changing a probability measure, namely introducing a drift. However, due to the orthogonality in (2.8) of the information premium, the information drift has a character which is not possible to reconstruct through a measure change, and thereby allows for additional flexibility in modelling. In the next section we shall look closer into this connection for a simple arithmetic two-factor model.

We have frequently referred to electricity as the prime example of a non-storable commodity. However, information premia are relevant in other markets as well, as already indicated. For example, weather markets often trades in forward contracts which are based on weather indices. These indices are obviously not tradeable, and hence not storable. Pricing of the contracts based on a conditional expectation of the future weather given current information $\mathcal{F}_t$ may be wrong, because weather forecasts are not accounted for. Weather forecasts give additional information that the market will include in the prices. Note that weather forecasts do not provide exact information about future weather conditions, but gives additional information about the future reducing the uncertainty. The markets for gas may be another example where the information premium could make sense. In gas markets there are a lot of information about production plans and hub storage capacities available that will affect future transactions but not necessarily how the market operates today. Gas is storable, of course, but we argue here the existence of an additional element of information premium as well.

2.2. **Some empirical evidence.** To provide empirical evidence for the existence of an information premium, we discuss the influence of CO2 emission costs on electricity forwards. We consider the market situation in the German EEX market autumn of 2007. The market knew at the time that from January 2008 there will be an introduction of CO2 emission right costs, and these would more or less explicitly influence the spot price of electricity. A significant portion of the electricity production in this market is coal and gas based, and the emission costs induced for these producers will be transfered over to the consumers. In the autumn of 2007, there was no such cost included in the spot prices, but the expectation was that around 60% of the CO2 emission price will be added to the electricity price. At the time, the CO2 price was 20 Euros, and thus an addition of approximately 12 Euros was expected for the spot price from January 2008.

Figure 1. Monthly base load forward prices from the EEX

In Fig. 1, we have plotted the monthly base load forward prices observed on the EEX on October 9 2007. The prices for the November and December contracts are significantly lower than the January and February 2008 contracts. We observe a large price increase from December 2007 to January 2008. The contract price increases from 47.2 Euro/MWh for the December contract, to 62.25 Euro/MWh for the January 2008 contract. The price raises by 15.05 Euro/MWh (about 32% increase). Some part of such an increase is naturally explained by the long Christmas holiday in December, and expected colder weather in January. However, the significant part of the price increase is due to the markets inclusion of CO2 prices. We see that this is not present in the November and December 2007 contracts.

To validate that the large price increase over the turn of the year has its origin in emission costs, we turn our attention to the quarterly base load contracts for 2008 and

Figure 2. Quarterly base load forward prices from the EEX

2009. In Fig. 2 we have plotted the prices of 8 quarterly contracts observed in the market on October 8 2007, starting with Q1 2008. Noteworthy here is the difference between Q4 2008 and Q1 2009. The price for Q4 2008 is 60.3 Euro/MWh, whereas the Q1 2009 contract costs 62.7 Euro/MWh, implying a price increase of 2.4 Euro/MWh. This clearly indicates that a large portion of the price jump observed for the monthly contracts from December to January is the market's reflection of the coming CO2 costs. In conclusion, we have a situation where the market explicitly takes into account future information of the spot price behavior when settling prices for forward contracts.

The EEX market does not have great flexibility in storing electricity to exploit higher prices. With such a flexibility, the spot prices would take the future costs into account and increase even before the CO2 emission costs have been introduced. To see this effect clearly, we look at the corresponding forward prices in the Nordic electricity market Nord Pool. The results we find here contrast clearly the difference between a market influenced largely by hydro power, and markets where electricity cannot easily be transported in time.

In the autumn of 2007, the water reservoir levels in the Nordic region were higher than average, and the producers had great flexibility in holding back their production to wait for the increasing prices expected in 2008. Looking at the monthly forward prices at Nord Pool on October 9 2007 reported in Fig. 3, we observe a very small price increase from the December 2007 to the January 2008 contract, relative to what we found at EEX. The prices were, respectively, 43.53 Euro/MWh and 49.9 Euro/MWh, leading to a price increase of 6.37 Euro/MWh (about 15% increase). Comparing this with the quarterly contracts (see Fig. 4), we find a mild increase from 2008 to 2009. In fact, the Q4 2008 contract costs 50.2 Euro/MWh and the Q1 2009 52.7 Euro/MWh, leading to an increase of 2.5 Euro/MWh. The quarterly contracts show a price difference over these two quarters on the same level as in the EEX market. We conclude that the flexibility to postpone production in the

Figure 3. Monthly forward prices from NordPool

Nordic market leads to an increase of prices much earlier, and the information effect is not so pronounced as in the EEX market, although observable.

Figure 4. Quarterly forward prices from NordPool

There are other typical examples where it is natural to let $\mathcal{G}_t$ be strictly greater than $\mathcal{F}_t$. Weather forecasts are of course heavily used in the electricity market as a basis for price formations, since cold and warm weather influences strongly the demand side. Further, weather predictions play a fundamental role in production planning, in particular for hydro power plants. Also, production plans themselves are a crucial factor for the forward prices. Such information is available in the market, and obviously taken into account when prices for future delivery of electricity is determined. On the other hand, since electricity is not storable, there is no reason why today's spot (and its information flow $\mathcal{F}_t$) should reflect these future events, since the spot today only is a result of today's supply and demand situation.

Yet another example is the effect of political decisions. The electricity and weather markets are still developing, and many political decisions are made concerning the legislation of these. For instance, the decision whether to build a new nuclear power plant within a market is not up to the producers alone, but also a political decision on national level. If a producer wishes to do so, the market knows immediately that the spot price may become significantly lower than the current expected level, since the supply side will be increased. On the other hand, if the application for building such a plant is turned down, this sends the price to higher levels. The same considerations hold for building connecting cables to other electricity markets, since this also alters the supply and demand side.

3. Forward pricing with future information

In this section we use the theory of enlargement of filtrations to derive forward prices in a concrete market model with future looking information. We consider a market where the spot price of electricity $S(t)$ evolves according to the following two-factor model:

$$S(t) = \Lambda(t) + X(t) + Y(t)\,. \tag{3.1}$$

Here $\Lambda(t)$ is a deterministic seasonality function, and $X(t)$ respectively $Y(t)$ are mean reverting factors following the dynamics

$$\begin{aligned} dX(t) &= -\alpha X(t)dt + \sigma dW(t)\,, \\ dY(t) &= -\beta Y(t)dt + dL(t)\,, \end{aligned}$$

with constant mean reversion parameters $\alpha, \beta > 0$. The processes $L(t)$ and $W(t)$ are independent, with L being a square integrable Lévy process and W a Brownian motion. Arithmetic multi factor models of this type are successfully capturing stylized features of electricity prices (see Meyer-Brandis and Tankov [30]) while at the same time they are more analytically tractable than alternative geometric models (see Benth, Kallsen and Meyer-Brandis [4]). In this setting, the base component $X(t)$ accounts for the long-term level of the price, while $Y(t)$ is the short-term spiky variations in the market. One could make the two sources of noise W and L, dependent (or correlated for $L = B$ a Brownian motion), however, to keep matters simple we refrain from doing this.

We let $\mathcal{F}_t$ be the filtration generated by the two noise processes W and L. Suppose now that the participants in the market have accessible additional information of the spot price behavior at some future time T_1. For a traded financial asset, this information would have an immediate impact on the price of the asset today. Due to the non-storability of electricity, the today's price of spot electricity, however, is unaffected by this information, and consequently the filtration $\mathcal{F}_t$ generated by the spot price factors misses to represent this essential part of information available to the market. In the following we are considering two specific situations of forward looking information available to the market and analyze the corresponding information premia (recall (2.7) for the definition). In the first situation the market has some idea about the underlying driving jump noise $L(T_1)$ at some time T_1, while in the second situation the market has knowledge about a third factor (temperature) which is correlated to the base component $X(t)$. Finally, we consider future knowledge of the base component mimicking the situation of CO2 emission costs. Obviously, these considerations are simplifications of the actual market situations. However, the examples provide some insight into how future information influence the price of forwards.

Before proceeding, let us include the predicted spot price with respect to $\mathcal{F}_t$ for completeness.

Proposition 3.1. *The predicted spot price based on $\mathcal{F}$ is given as*

$$f_{\mathcal{F}}(t,T) = \Lambda(T) + X(t)e^{-\alpha(T-t)} + Y(t)e^{-\beta(T-t)} - \frac{i\psi'(0)}{\beta}(1 - e^{-\beta(T-t)}),$$

with $\psi(\theta)$ being the cumulant function of $L(1)$.

Proof. We have that

$$X(T) = X(t)\mathrm{e}^{-\alpha(T-t)} + \int_t^T \sigma\mathrm{e}^{-\alpha(T-s)}\,dW(s),$$

and

$$Y(T) = Y(t)\mathrm{e}^{-\beta(T-t)} + \int_t^T \mathrm{e}^{-\beta(T-s)}\,dL(s).$$

Hence, by using measurability of $X(t)$ and $Y(t)$ with respect to to $\mathcal{F}_t$ and the independent increment property of the Lévy process L and Brownian motion W, the result follows. □

3.1. **Forward looking information about jump noise.** Let the filtration $\mathcal{H}_t$ be defined as

$$\mathcal{H}_t = \mathcal{F}_t \vee \sigma(L(T_1)),$$

that is $\mathcal{H}_t$ represents the complete knowledge about the value of the jump noise $L(T_1)$ at some future time T_1 in addition to the information contained in $\mathcal{F}_t$. In this first situation we assume the market has some information, not necessary complete, about the jump noise $L(T_1)$. That is, the information filtration available to the market, denoted by $\mathcal{G}_t$, is such that

$$\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{H}_t. \tag{3.2}$$

Note that $\mathcal{G}_t = \mathcal{F}_t$ whenever $t \geq T_1$. Hence, the information premium is equal to zero for all times $t \leq T$ with $t \geq T_1$, natural in view of the fact that in this case the "additional" information is no longer relevant.

Proposition 3.2. *Suppose the market has available the information described by the filtration $\mathcal{G}_t$ in (3.2). If $t \leq T \leq T_1$, then the information premium is given by*

$$I_{\mathcal{G}}(t,T) = \frac{1}{\beta}\left(\frac{\mathbb{E}\left[L(T_1) - L(t)\,|\,\mathcal{G}_t\right]}{T_1 - t} - (-i\psi'(0))\right)\left(1 - e^{-\beta(T-t)}\right), \tag{3.3}$$

with $\psi(\theta)$ being the cumulant function of $L(1)$. Furthermore, for $t \leq T_1 \leq T$ we get

$$I_{\mathcal{G}}(t,T) = e^{-\beta(T-T_1)} I_{\mathcal{G}}(t,T_1)\,. \tag{3.4}$$

Before we proceed to proof Prop. 3.2 we restate the following well known result about the information yield in this situation (see Thm. 3 on p. 356 in Protter [31] and Prop. 18 in Di Nunno et al. [16]):

Lemma 3.3. *Let the filtration $\mathcal{G}_t$ be as in* (3.2)*. Then*

$$L(t) - \int_0^t \frac{\mathbb{E}\left[L(T_1) - L(s)\,|\,\mathcal{G}_s\right]}{(T_1 - s)}\,ds$$

is a $\mathcal{G}_t$-martingale on $[0, T_1]$.

Proof. (*Prop. 3.2*) Let first $T \leq T_1$. Since $X(T)$ is independent of $L(t)$, conditioning $X(T)$ on $\mathcal{G}_t$ coincides with $\mathcal{F}_t$. Hence, by the definition of $I_{\mathcal{G}}(t,T)$, we calculate,

$$\begin{aligned}
I_{\mathcal{G}}(t,T) &= \mathbb{E}\left[Y(T)\,|\,\mathcal{G}_t\right] - \mathbb{E}\left[Y(T)\,|\,\mathcal{F}_t\right] \\
&= \mathbb{E}\left[Y(t)\mathrm{e}^{-\beta(T-t)} + \int_t^T \mathrm{e}^{-\beta(T-s)}\,dL(s)\,|\,\mathcal{G}_t\right] \\
&\quad - \mathbb{E}\left[Y(t)\mathrm{e}^{-\beta(T-t)} + \int_t^T \mathrm{e}^{-\beta(T-s)}\,dL(s)\,|\,\mathcal{F}_t\right] \\
&= \mathbb{E}\left[\int_t^T \mathrm{e}^{-\beta(T-s)}\,dL(s)\,|\,\mathcal{G}_t\right] + \frac{\mathrm{i}\psi'(0)}{\beta}(1 - \mathrm{e}^{-\beta(T-t)})\,,
\end{aligned}$$

where we have used $\mathcal{F}_t$ and $\mathcal{G}_t$ measurability of $Y(t)$ in the last equality. Now, by Lemma 3.3

$$\begin{aligned}
\mathbb{E}\left[\int_t^T \mathrm{e}^{-\beta(T-s)}\,dL(s)\,|\,\mathcal{G}_t\right] &= \mathbb{E}\left[\int_t^T \mathrm{e}^{-\beta(T-s)}\frac{E\left[L(T_1)\,|\,\mathcal{G}_s\right] - L(s)}{T_1 - s}\,ds\,|\,\mathcal{G}_t\right] \\
&= \int_t^T \frac{\mathrm{e}^{-\beta(T-s)}}{T_1 - s} E\left[L(T_1) - L(s)\,|\,\mathcal{G}_t\right]\,ds\,.
\end{aligned}$$

By Proposition A.3, and Remark A.4 we compute with $g(s) = \frac{1}{T_1 - s}$ and $f(s) = 1$ ($t \leq s \leq T$)

$$E\left[L(T_1) - L(s)\,|\,\mathcal{G}_t\right] = \frac{T_1 - s}{T_1 - t}\,E\left[L(T_1) - L(t)\,|\,\mathcal{G}_t\right]\,.$$

Thus

$$\begin{aligned}
\mathbb{E}\left[\int_t^T \mathrm{e}^{-\beta(T-s)}\,dL(s)\,|\,\mathcal{G}_t\right] &= \int_t^T \frac{\mathrm{e}^{-\beta(T-s)}}{T_1 - s}\frac{T_1 - s}{T_1 - t}\,\mathbb{E}\left[L(T_1) - L(t)\,|\,\mathcal{G}_t\right]\,ds \\
&= \frac{\mathbb{E}\left[L(T_1) - L(t)\,|\,\mathcal{G}_t\right]}{T_1 - t}\int_t^T \mathrm{e}^{-\beta(T-s)}\,ds \\
&= \frac{\mathbb{E}\left[L(T_1)\,|\,\mathcal{G}_t - L(t)\right]}{(T_1 - t)\beta}\left(1 - \mathrm{e}^{-\beta(T-t)}\right)\,,
\end{aligned}$$

which proofs the result for $T \leq T_1$. If $T > T_1$ we decompose

$$I_{\mathcal{G}}(t,T) = \mathbb{E}\left[Y(T_1) + \mathbb{E}\left[Y(T) - Y(T_1)\,|\,\mathcal{F}_{T_1}\right]\,|\,\mathcal{G}_t\right] - \mathbb{E}\left[Y(T_1) - \mathbb{E}\left[Y(T) - Y(T_1)\,|\,\mathcal{F}_{T_1}\right]\,|\,\mathcal{F}_t\right].$$

A computation yields

$$\mathbb{E}\left[Y(T) - Y(T_1)\,|\,\mathcal{F}_{T_1}\right] = Y(T_1)(\mathrm{e}^{-\beta(T-T_1)} - 1) - \frac{\mathrm{i}\psi'(0)}{\beta}(1 - \mathrm{e}^{-\beta(T-T_1)}),$$

and thus,

$$I_{\mathcal{G}}(t,T) = \mathbb{E}\left[Y(T_1)\mathrm{e}^{-\beta(T-T_1)}\,|\,\mathcal{G}_t\right] - \mathbb{E}\left[Y(T_1)\mathrm{e}^{-\beta(T-T_1)}\,|\,\mathcal{F}_t\right] = \mathrm{e}^{-\beta(T-T_1)} I_{\mathcal{G}}(t,T_1).$$

The Proposition follows. □

Let us consider the situation where the market knows whether the value of $L(T_1)$ is above or below a given threshold, say K. In this case, the filtration $\mathcal{G}_t$ is specified as

$$\mathcal{G}_t = \mathcal{F}_t \vee \sigma(\mathbf{1}_{\{L(T_1)\leq K\}}).$$

where $\mathbf{1}_A(\omega)$ is the indicator function in ω on a set $A \subseteq \Omega$. For example, the market may have the information that an important producer will have an outage (due to maintenance, say). The network will then experience a sudden drop in supply, which will lead to an increase in spot prices at time T_1. We can model such a situation by saying that $L(T_1) \geq K$. Another relevant situation is when a new transport line for electricity is opened. This will lead to an increase in supply, which we can model as $L(T_1) \leq K$, since the possibility for additional supply of electricity into the network will push prices down. Similarily, the decision to build a new power plant will have the same effect.

We analyse the information premium for this specification of $\mathcal{G}_t$. If Z is a random variable with the same distribution as the increment $L(T_1) - L(t)$, we find

$$\mathbb{E}\left[L(T_1) - L(t)\,|\,\mathcal{G}_t\right] = \mathbb{E}\left[Z\,|\,Z \leq K - L(t)\right]\mathbf{1}_{\{L(T_1)\leq K\}} + \mathbb{E}\left[Z\,|\,Z > K - L(t)\right]\mathbf{1}_{\{L(T_1)>K\}}.$$

Thus, using the fact that $\mathbb{E}[Z] = -\mathrm{i}\psi'(0)(T_1 - t)$, we find that the information premium can be expressed as

$$\begin{aligned} I_{\mathcal{G}}(t,T) = \frac{1 - \mathrm{e}^{-\beta(T-t)}}{\beta(T_1 - t)} &\left\{(\mathbb{E}[Z\,|\,Z \leq K - L(t)] - \mathbb{E}[Z])\,\mathbf{1}_{\{L(T_1)\leq K\}}\right. \\ &\left. + (\mathbb{E}[Z\,|\,Z > K - L(t)] - \mathbb{E}[Z])\,\mathbf{1}_{\{L(T_1)>K\}}\right\}. \end{aligned}$$

If we know that $L(T_1) > K$, we see that the information premium becomes positive. On the other hand, the information $L(T_1) \leq K$ leads to a negative information premium. In both cases we have a quantification of the premium in terms of the information available and the model parameters. These conclusions are in line with the market heuristics, with the first situation comparing to a production outage with the consequence of lower supply, and the latter of the increase of supply through new cables connecting markets, say.

Note that the information premium tends to zero whenever $T \to \infty$ and T_1 fixed. This follows from (3.4). The impact of information at time T_1 on contracts maturing far into the future (that is, $T \gg T_1$) will be very small, which is natural from a practical point of view. However, the sign of the premium will remain the same for all contracts with $T > T_1$.

If $L = \xi\widetilde{W}$, with $\widetilde{W}$ a Brownian motion and ξ a positive constant, we can calculate the conditional expectations explicitly. In this case it follows that

$$I_{\mathcal{G}}(t,T) = \frac{\xi(1-\mathrm{e}^{-\beta(T-t)})}{\beta\sqrt{T_1-t}}\phi\left(\frac{K-\widetilde{W}(t)}{\xi\sqrt{T_1-t}}\right)\left\{\frac{\mathbf{1}_{\{\widetilde{W}(T_1)>K/\xi\}}}{1-\Phi\left(\frac{K-\widetilde{W}(t)}{\xi\sqrt{T_1-t}}\right)}-\frac{\mathbf{1}_{\{\widetilde{W}(T_1)\leq K/\xi\}}}{\Phi\left(\frac{K-\widetilde{W}(t)}{\xi\sqrt{T_1-t}}\right)}\right\}.$$

Here, Φ is the cumulative standard normal distribution function and ϕ its density. We note in this, and the more general expression above, that the value and sign of the information premium will vary stochastically with $L(t)$.

3.2. **Future information about correlated temperature.** Assume that the temperature follows a seasonal Ornstein-Uhlenbeck process

$$dZ(t) = -\gamma(Z(t)-\mu(t))\,dt + \eta\,dB(t)\,, \tag{3.5}$$

where B Brownian motion, independent of W, and $\mu(t)$ is the deterministic seasonal level to which the temperature is mean-reverting with rate $\gamma > 0$. The temperature "volatility" η is assumed to be a positive constant. In Benth and Šaltytė-Benth [7], it is shown that this model fits daily temperature observations in Stockholm, Sweden, reasonably well, especially with the volatility η being a seasonal function (see also Benth and Šaltytė-Benth [6] and Benth, Šaltytė Benth and Koekebakker [8]). Here we suppose it to be constant for simplicity. Further, we assume that $\mu(t)$ is a bounded and continuous function. The spot price is correlated with the temperature. More precisely, we here suppose that the base component $X(t)$ has a dynamics defined as

$$dX(t) = -\alpha X(t)\,dt + \sigma\rho\,dB(t) + \sigma\sqrt{1-\rho^2}\,dW(t)\,. \tag{3.6}$$

Observe that the base component X of the spot price and temperature are correlated with a correlation coefficient ρ.

In the Nordic electricity market Nord Pool, where the main driver of electricity demand is temperature due to heating in the winter, it is to be expected that the correlation is negative, since lower temperatures implies higher prices due to increasing demand for heating. On the other hand, higher temperatures means that households are decreasing their demand for heating, meaning that prices should go down. For markets being dominated by high temperatures in the summer, and moderate in the winter season (like for instance in California), the correlation may be positive due to increasing demand for air conditioning cooling in warm periods.

Let us suppose that the market has accessible some future information about the temperature at time T_1. Again, complete knowledge of the temperature $Z(T_1)$ would correspond to the enlarged filtration

$$\mathcal{H}_t \triangleq \mathcal{F}_t \vee \sigma(Z(T_1))\,, \tag{3.7}$$

where we now assume $\mathcal{F}_t$ to be the filtration generated by W, B, and L. Since

$$Z(T_1) = Z(0)\mathrm{e}^{-\gamma T_1} + \int_0^{T_1}\gamma\mu(s)\mathrm{e}^{-\gamma(T_1-s)}\,ds + \eta\mathrm{e}^{-\gamma T_1}\int_0^{T_1}\mathrm{e}^{\gamma s}\,dB(s)\,,$$

we have that

$$\sigma(Z(T_1)) = \sigma(\int_0^{T_1} \mathrm{e}^{\gamma s}\, dB(s))\,. \tag{3.8}$$

We assume that the market has access to some information about $Z(T_1)$ additionally to the information contained in $\mathcal{F}_t$, that is, the information is represented by some filtration $\mathcal{G}_t$ such that

$$\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{H}_t\,. \tag{3.9}$$

We can now apply Thm. A.1 to identify the information yield for B and to construct a new Brownian motion with respect to the filtration $\mathcal{G}_t$.

Proposition 3.4. *Let the filtration $\mathcal{G}_t$ be as in* (3.2)*. Then*

$$B(t) - \int_0^t a(s)\mathbb{E}\left[\int_s^{T_1} e^{\gamma u}\, dB(u)\,|\,\mathcal{G}_s\right] ds\,,$$

is a $\mathcal{G}_t$-Brownian motion on $[0, T_1]$*, with*

$$a(t) = \frac{2\gamma e^{\gamma t}}{e^{2\gamma T_1} - e^{2\gamma t}}\,.$$

Proof. With the notation of Thm. A.1, we find with $U(t) = B(t)$ and $G = \int_0^{T_1} \mathrm{e}^{\gamma s}\, dB(s)$

$$\rho(t) = \mathbb{E}\left[B(t)\int_0^{T_1} \mathrm{e}^{\gamma s}\, dB(s)\right] = \frac{1}{\gamma}(\mathrm{e}^{\gamma t} - 1)\,,$$

and

$$\tau = \mathbb{E}\left[\left(\int_0^{T_1} \mathrm{e}^{\gamma s}\, dB(s)\right)^2\right] = \frac{1}{2\gamma}(\mathrm{e}^{2\gamma T_1} - 1)\,.$$

Furthermore, for $0 \le s \le T_1$

$$b_t(s) = \gamma \mathrm{e}^{\gamma s}\int_s^t \frac{2\gamma \mathrm{e}^{\gamma v}}{\mathrm{e}^{2\gamma T_1} - \mathrm{e}^{2\gamma v}}\, dv = \gamma \mathrm{e}^{\gamma s}\int_s^t a(v)\, dv\,.$$

From Thm. A.1 and Prop. A.2 we have that

$$B(t) - \int_0^t b_t(s)B(s)\, ds - \int_0^t a(s)(\mathbb{E}[G\,|\,\mathcal{G}_s] - \rho'(s)B(s))\, ds\,. \tag{3.10}$$

is $\mathcal{G}_t$-Brownian motion. We investigate the two last terms in more detail:

Consider the first integral: Since $b_t(t) = 0$, it holds that

$$\begin{aligned}
\int_0^t b_t(s)B(s)\, ds &= \int_0^t b_s(s)B(s)\, ds + \int_0^t \int_0^s \frac{\partial b_s}{\partial s}(u)B(u)\, du\, ds \\
&= \int_0^t \int_0^s \frac{\partial b_s}{\partial s}(u)B(u)\, du\, ds \\
&= \gamma\int_0^t a(s)\int_0^s \mathrm{e}^{\gamma u}B(u)\, du\, ds\,.
\end{aligned}$$

By Itô's Formula, we find

$$\mathrm{e}^{\gamma s}B(s) = \gamma \int_0^s \mathrm{e}^{\gamma u}B(u)\,du + \int_0^s \mathrm{e}^{\gamma u}\,dB(u)\,,$$

and therefore the first integral term becomes

$$\int_0^t b_t(s)B(s)\,ds = \int_0^t a(s)\left(\mathrm{e}^{\gamma s}B(s) - \int_0^s \mathrm{e}^{\gamma u}\,dB(u)\right)\,ds\,.$$

We find the second integral to be

$$\int_0^t a(s)\left(\mathbb{E}[G\,|\,\mathcal{G}_s] - \rho'(s)B(s)\right)\,ds = \int_0^t a(s)\left(\mathbb{E}[\int_0^{T_1} \mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_s] - \mathrm{e}^{\gamma s}B(s)\right)\,ds\,.$$

Collecting terms, we end up with

$$B(t) - \int_0^t a(s)\mathbb{E}\left[\int_s^{T_1} \mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_s\right]\,ds\,.$$

Thus, the Proposition is proved. □

We obtain from the Proposition above an expression for the information drift defined in Def. 2.4, being

$$\theta(t) = a(t)\mathbb{E}\left[\int_t^{T_1} \mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_t\right]\,. \tag{3.11}$$

Observe that $\theta(t)$ is not $\mathcal{F}_t$ adapted since we condition on the bigger filtration $\mathcal{G}_t$. Hence, we can not associate this information drift to any equivalent measure change. Since

$$\int_t^{T_1} \mathrm{e}^{\gamma u}\,dB(u) = \frac{1}{\eta}\left(\mathrm{e}^{\gamma T_1}Z(T_1) - \mathrm{e}^{\gamma t}Z(t) - \gamma\int_t^{T_1} \mu(u)\mathrm{e}^{\gamma u}\,du\right)\,, \tag{3.12}$$

we have that the information drift in (3.11) can be expressed as

$$\theta(t) = \frac{a(t)}{\eta}\left(\mathrm{e}^{\gamma T_1}\mathbb{E}[Z(T_1)\,|\,\mathcal{G}_t] - \mathrm{e}^{\gamma t}Z(t) - \gamma\int_t^{T_1} \mu(u)\mathrm{e}^{\gamma u}\,du\right)\,.$$

Thus, the information drift depends (not surprisingly) on the temperatures at time t and T_1 (the latter conditionally on $\mathcal{G}_t$), in addition to the speed of mean-reversion γ, the temperature "volatility" η and the weighted average of the seasonality function μ up to time to information T_1.

Having identified the information drift, we are now able to determine the corresponding information premium.

Proposition 3.5. *Suppose the market has some information at time t about the future temperature $Z(T_1)$ represented through the filtration $\mathcal{G}_t$ in* (3.9). *If $t \leq T \leq T_1$, the information premium is given by*

$$I_{\mathcal{G}}(t,T) = V(t,T)\left(e^{\gamma T_1}\mathbb{E}[Z(T_1)\,|\,\mathcal{G}_s] - e^{\gamma t}Z(t) - \gamma\int_t^{T_1} \mu(u)e^{\gamma u}\,du\right)\,, \tag{3.13}$$

where

$$V(t,T)=\rho\,\frac{2\gamma\sigma e^{\gamma T}\left(1-e^{-(\alpha+\gamma)(T-t)}\right)}{\eta(\alpha+\gamma)\left(e^{2\gamma T_1}-e^{2\gamma t}\right)}\,. \tag{3.14}$$

For $t\le T_1\le T$ we get

$$I_{\mathcal{G}}(t,T)=e^{-\alpha(T-T_1)}I_{\mathcal{G}}(t,T_1)\,. \tag{3.15}$$

Proof. Consider first $T\le T_1$. Since Y is independent of B, we get as before by the definition of $I_{\mathcal{G}}(t,T)$,

$$\begin{aligned}
I_{\mathcal{G}}(t,T)&=\mathbb{E}\left[X(T)\,|\,\mathcal{G}_t\right]-\mathbb{E}\left[X(T)\,|\,\mathcal{F}_t\right]\\
&=\mathbb{E}\left[X(t)\mathrm{e}^{-\alpha(T-t)}+\sigma\rho\int_t^T\mathrm{e}^{-\alpha(T-s)}\,dB(s)+\sigma\sqrt{1-\rho^2}\int_t^T\mathrm{e}^{-\alpha(T-s)}\,dW(s)\,|\,\mathcal{G}_t\right]\\
&\quad-\mathbb{E}\left[X(t)\mathrm{e}^{-\alpha(T-t)}+\sigma\rho\int_t^T\mathrm{e}^{-\alpha(T-s)}\,dB(s)+\sigma\sqrt{1-\rho^2}\int_t^T\mathrm{e}^{-\alpha(T-s)}\,dW(s)\,|\,\mathcal{F}_t\right]\\
&=\sigma\rho\mathbb{E}\left[\int_t^T\mathrm{e}^{-\alpha(T-s)}\,dB(s)\,|\,\mathcal{G}_t\right]\,,
\end{aligned}$$

where we have used $\mathcal{F}_t$ and $\mathcal{G}_t$ measurability of $X(t)$, the independence of W and B, and the $\mathcal{F}_t$-martingale property of B in the last equality. Now, by Prop. 3.4

$$\begin{aligned}
\mathbb{E}\left[\int_t^T\mathrm{e}^{-\alpha(T-s)}\,dB(s)\,|\,\mathcal{G}_t\right]&=\mathbb{E}\left[\int_t^T a(s)\mathrm{e}^{-\alpha(T-s)}\mathbb{E}\left[\int_s^{T_1}\mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_s\right]\,ds\,|\,\mathcal{G}_t\right]\\
&=\int_t^T a(s)\mathrm{e}^{-\alpha(T-s)}\mathbb{E}\left[\int_s^{T_1}\mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_t\right]\,ds\,.
\end{aligned}$$

By Proposition A.3 with $g(s)=a(s)$ and $f(s)=\mathrm{e}^{\gamma s}$,

$$\mathbb{E}\left[\int_s^{T_1}\mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_t\right]=\mathbb{E}\left[\int_t^{T_1}\mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_t\right]\frac{\mathrm{e}^{2\gamma T_1}-\mathrm{e}^{2\gamma s}}{\mathrm{e}^{2\gamma T_1}-\mathrm{e}^{2\gamma t}}\,.$$

Thus,

$$\begin{aligned}
\mathbb{E}\left[\int_t^T\mathrm{e}^{-\alpha(T-s)}\,dB(s)\,|\,\mathcal{G}_t\right]&=\frac{\mathbb{E}\left[\int_t^{T_1}\mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_t\right]}{\mathrm{e}^{2\gamma T_1}-\mathrm{e}^{2\gamma t}}\int_t^T a(s)\mathrm{e}^{-\alpha(T-s)}(\mathrm{e}^{2\gamma T_1}-\mathrm{e}^{2\gamma s})\,ds\\
&=\frac{2\gamma\rho\sigma\mathbb{E}\left[\int_t^{T_1}\mathrm{e}^{\gamma u}\,dB(u)\,|\,\mathcal{G}_t\right]}{(\alpha+\gamma)\left(\mathrm{e}^{2\gamma T_1}-\mathrm{e}^{2\gamma t}\right)}\mathrm{e}^{\gamma T}\left(1-\mathrm{e}^{-(\alpha+\gamma)(T-t)}\right)\,,
\end{aligned}$$

which, after appealing to (3.12), proofs the result for $T\le T_1$. If $T>T_1$ we decompose as in the proof of Prop. 3.2

$$\begin{aligned}
I_{\mathcal{G}}(t,T)=&\,\mathbb{E}\left[X(T_1)+\mathbb{E}\left[X(T)-X(T_1)\,|\,\mathcal{F}_{T_1}\right]\,|\,\mathcal{G}_t\right]\\
&-\mathbb{E}\left[X(T_1)-\mathbb{E}\left[X(T)-X(T_1)\,|\,\mathcal{F}_{T_1}\right]\,|\,\mathcal{F}_t\right]\,.
\end{aligned}$$

We compute

$$\mathbb{E}\left[X(T)-X(T_1)\,|\,\mathcal{F}_{T_1}\right]=X(T_1)(\mathrm{e}^{-\alpha(T-T_1)}-1)\,,$$

and get

$$\begin{aligned} I_{\mathcal{G}}(t,T) &= \mathbb{E}\left[X(T_1)\mathrm{e}^{-\alpha(T-T_1)} \,|\, \mathcal{G}_t\right] - \mathbb{E}\left[X(T_1)\mathrm{e}^{-\alpha(T-T_1)} \,|\, \mathcal{F}_t\right] \\ &= \mathrm{e}^{-\alpha(T-T_1)} I_{\mathcal{G}}(t,T_1)\,. \end{aligned}$$

Hence, the proof is complete. □

If we assume that we have an exact temperature forecast for time T_1, i.e. $\mathcal{G}_t = \mathcal{H}_t$, then the information premium depends explicitly on the temperatures at time t and T_1:

$$I_{\mathcal{H}}(t,T) = V(t,T)\left(\mathrm{e}^{\gamma T_1} Z(T_1) - \mathrm{e}^{\gamma t} Z(t) - \gamma \int_t^{T_1} \mu(u)\mathrm{e}^{\gamma u}\,du\right). \tag{3.16}$$

We next investigate the sign of the information premium using (3.16).

First, observe that the sign of $V(t,T)$ in (3.14) is completely determined by the correlation coefficient ρ. In the Nord Pool market, we expect ρ to be negative, and hence V becomes negative as well. In electricity markets where demand for cooling in the summer is high, like the Californian market say, we expect ρ to be positive. In this case V is positive. Assume now that the weather forecast predicts a temperature decrease, in the sense that,

$$Z(T_1) < \mathrm{e}^{-\gamma(T_1-t)} Z(t) + \gamma \int_t^{T_1} \mu(u)\mathrm{e}^{-\gamma(T_1-u)}\,du\,.$$

Such a knowledge of the future temperature will imply a negative information premium in the Nord Pool market. This is in line with the economical reasoning that consumers will hedge the expected future price increase due to increasing demand, and thereby willing to pay a premium to the producers. The situation is turned around if the temperature is forecasted to increase. In that case the producers face declining prices, and will be willing to pay a premium for hedging their future production, thus implying a negative information premium. We have an explanation of the stochastic change of the sign of the information premium in terms of temperature forecasts, that is, demand forecast.

If we are in (the more realistic) situation of having temperature forecasts at several future time points $T_1 \leq ... \leq T_m$, we can use Prop. 3.5 recursively to determine the corresponding information premium. Let us introduce the notation

$$\mathcal{H}_t \triangleq \mathcal{F}_t \vee \sigma(Z(T_1), ..., Z(T_m)) \tag{3.17}$$

$$\mathcal{H}_t^i \triangleq \mathcal{F}_t \vee \sigma(Z(T_i))\,, \quad i = 1, ..., m\,. \tag{3.18}$$

Proposition 3.6. *Suppose the market has available at time t temperature forecasts for $Z(T_1), ..., Z(T_m)$ represented through the filtration $\mathcal{H}_t$ in (3.17). Then the information premium is given as*

(3.19)
$$\begin{cases} I_{\mathcal{H}}(t,T) = \sum_{j=1}^{i-1} I_{\mathcal{H}^j}(T_{j-1},T_j) + I_{\mathcal{H}^j}(T_{i-1},T)\,; & T_{i-1} \leq T \leq T_i\,; \quad i = 1, ..., m. \\ I_{\mathcal{H}}(t,T) = e^{-\alpha(T-T_m)} I_{\mathcal{H}}(t,T_m)\,; & T_m < T\,. \end{cases}$$

where we set $T_0 := t$ and where $I_{\mathcal{H}^j}(T_{j-1},T_j)$ is the expression for the information premium in Prop. 3.5 with $\mathcal{G} = \mathcal{H}^j$, $t = T_{j-1}$, and $T = T_j$.

Proof. For notational simplicity we only consider the case $m = 2$. The general proof follows analogously.

First, note that due to (3.8) we have

$$\begin{aligned}\mathcal{H}_t &= \mathcal{F}_t \vee \sigma\left(\int_0^{T_1} \mathrm{e}^{\gamma s}\, dB(s), \int_0^{T_2} \mathrm{e}^{\gamma s}\, dB(s)\right) \\ &= \mathcal{F}_t \vee \sigma\left(\int_0^{T_1} \mathrm{e}^{\gamma s}\, dB(s), \int_{T_1}^{T_2} \mathrm{e}^{\gamma s}\, dB(s)\right), \end{aligned} \tag{3.20}$$

and

$$\mathcal{H}_t^i = \mathcal{F}_t \vee \sigma\left(\int_0^{T_i} \mathrm{e}^{\gamma s}\, dB(s)\right), \quad i = 1, 2\,.$$

Recall from the proof of Prop. 3.5 that the information premium is given as

$$I_{\mathcal{H}}(t,T) = \mathbb{E}\left[X(T)\,|\,\mathcal{H}_t\right] - \mathbb{E}\left[X(T)\,|\,\mathcal{F}_t\right]\,.$$

For $t \leq T \leq T_1$, the stochastic integral $\int_{T_1}^{T_2} \mathrm{e}^{\gamma s}\, dB(s)$ is independent of $X(T)$, and hence $\mathbb{E}\left[X(T)\,|\,\mathcal{H}_t\right] = \mathbb{E}\left[X(T)\,|\,\mathcal{H}_t^1\right]$. Therefore, using Prop. 3.5, it follows that

$$I_{\mathcal{H}}(t,T) = \mathbb{E}\left[X(T)\,|\,\mathcal{H}_t^1\right] - \mathbb{E}\left[X(T)\,|\,\mathcal{F}_t\right] = I_{\mathcal{H}^1}(t,T)\,.$$

Next, assume $T_1 \leq T \leq T_2$. Then

$$\begin{aligned}
&\mathbb{E}\left[X(T)\,|\,\mathcal{H}_t\right] - \mathbb{E}\left[X(T)\,|\,\mathcal{F}_t\right] \\
&\quad = \mathbb{E}\left[X(t)\mathrm{e}^{-\alpha(T-t)} + \sigma\rho\int_t^{T_1} \mathrm{e}^{-\alpha(T_1-s)}\, dB(s) + \sigma\rho\int_{T_1}^{T} \mathrm{e}^{-\alpha(T-s)}\, dB(s)\,|\,\mathcal{H}_t\right] \\
&\qquad - \mathbb{E}\left[X(t)\mathrm{e}^{-\alpha(T-t)} + \sigma\rho\int_t^{T} \mathrm{e}^{-\alpha(T-s)}\, dB(s)\,|\,\mathcal{F}_t\right] \\
&\quad = \sigma\rho\mathbb{E}\left[\int_t^{T_1} \mathrm{e}^{-\alpha(T_1-s)}\, dB(s)\,|\,\mathcal{H}_t^1\right] + \sigma\rho\mathbb{E}\left[\mathbb{E}\left[\int_{T_1}^{T} \mathrm{e}^{-\alpha(T-s)}\, dB(s)\,|\,\mathcal{H}_{T_1}^2\right]\,|\,\mathcal{H}_t\right] \\
&\quad = I_{\mathcal{H}^1}(t,T_1) + \mathbb{E}\left[I_{\mathcal{H}^2}(T_1,T)\,|\,\mathcal{H}_t\right]\,,
\end{aligned}$$

where in the second to the last equation we have used the independence of $\int_t^{T_1} \mathrm{e}^{-\gamma(T_1-s)}\, dB(s)$ and $\int_{T_1}^{T_2} \mathrm{e}^{\gamma s}\, dB(s)$ together with observation (3.20) for the first term, and the fact that $\mathcal{H}_{T_1} = \mathcal{H}_{T_1}^2$ for the second term. The last equation follows from Prop. 3.5. Finally, from expression (3.13) and again observation (3.20), it follows that $I_{\mathcal{H}^2}(T_1,T)$ is $\mathcal{H}_t$ measurable and $\mathbb{E}\left[I_{\mathcal{H}^2}(T_1,T)\,|\,\mathcal{H}_t\right] = I_{\mathcal{H}^2}(T_1,T)$

For $T_2 < T$, the reasoning is as in the proof of Prop. 3.5 (final part). □

Indeed, by considering different maturities T, we can obtain a change in sign of the information premium based on temperature (that is, demand) forecasts.

3.3. **CO2 emission prices and electricity.** Let us return to the case of CO2 emission cost and the effect on electricity prices at the EEX as we discussed in Section 2. We consider again the spot model in Subsect. 3.1. One way to interpret the effect of CO2 emission costs to electricity spot prices is to assume that it influences the stochastic mean level $X(t)$. Suppose that complete knowledge of the mean level $X(T_1)$ would correspond to the enlarged filtration

$$\mathcal{H}_t \triangleq \mathcal{F}_t \vee \sigma(X(T_1)) \,, \tag{3.21}$$

where we now let $\mathcal{F}_t$ be the filtration generated by W and L. Since

$$X(T_1) = X(0)\mathrm{e}^{-\alpha T_1} + \sigma \mathrm{e}^{-\alpha T_1} \int_0^{T_1} \mathrm{e}^{\alpha s}\, dW(s) \,,$$

we have that

$$\sigma(X(T_1)) = \sigma\Big(\int_0^{T_1} \mathrm{e}^{\alpha s}\, dW(s)\Big) \,. \tag{3.22}$$

We assume that the market has access to some information about $X(T_1)$ additionally to the information contained in $\mathcal{F}_t$, that is, the information is represented by some filtration $\mathcal{G}_t$ such that

$$\mathcal{F}_t \subset \mathcal{G}_t \subset \mathcal{H}_t \,. \tag{3.23}$$

We can now basically follow the derivations in Subsect. 3.2, which indeed is analogous to the situation we are in. The information drift (see Prop. 3.4) becomes

$$\theta(t) = a(t)\mathbb{E}\left[\int_t^{T_1} \mathrm{e}^{\alpha u}\, dW(u) \,|\, \mathcal{G}_t\right] \,, \tag{3.24}$$

where

$$a(t) = \frac{2\alpha \mathrm{e}^{\alpha t}}{\mathrm{e}^{2\alpha T_1} - \mathrm{e}^{2\alpha t}} \,. \tag{3.25}$$

Likewise, we can represent the information drift in terms of the level X,

$$\theta(t) = \frac{a(t)}{\sigma} \left\{ \mathrm{e}^{\alpha T_1} \mathbb{E}\left[X(T_1) \,|\, \mathcal{G}_t\right] - \mathrm{e}^{\alpha t} X(t) \right\} \,. \tag{3.26}$$

Furthermore, a similar calculation as in Prop. 3.5 gives the information premium

$$I_{\mathcal{G}}(t,T) = \mathrm{e}^{\alpha(T_1 - T)} \frac{\mathrm{e}^{2\alpha T} - \mathrm{e}^{2\alpha t}}{\mathrm{e}^{2\alpha T_1} - \mathrm{e}^{2\alpha t}} \left\{ \mathbb{E}\left[X(T_1) \,|\, \mathcal{G}_t\right] - \mathrm{e}^{-\alpha(T_1 - t)} X(t) \right\} \,, \tag{3.27}$$

for $T \leq T_1$. If $T > T_1$, we have $I_{\mathcal{G}}(t,T) = \mathrm{e}^{-\alpha(T - T_1)} I_{\mathcal{G}}(t, T_1)$.

Knowing that the CO2 emission prices will affect the spot price of electricity can be modelled as knowing the stochastic mean level at time T_1, corresponding to complete information. Thus, if we are at time t before the emission prices has come into play, we know that $X(T_1) > X(t)$, and therefore we get a positive information premium. This is indeed what we observed in the EEX market (recall the discussion in Sect. 2).

Alternatively, we may interpret the introduction of CO2 pricing as the knowledge that $X(T_1)$ is above a level K. This will lead us back to similar considerations as in Subsect. 3.1. We leave the details to the reader.

4. Conclusions and future research

We have demonstrated that thederivation of forward prices on non-storable commodities using only information generated by the spot price is fundamentally wrong. Such an approach is unable to take into account forward looking information, typically being weather forecasts, outages and new market constraints like the introduction of CO2 emission fees. In this paper we explicitly include such information in the forward price dynamics.

We introduce the notion of information premium as the premium charged for including forward looking market information. When measuring the risk premium, which is the difference between the forward price and the predicted spot price, under the wrong information assumption, the information premium will be one component in the risk premium. Thus, supposedly irregular behavior of measured risk premia might in reality be due to information miss-specification in the model. In this sense, we show in some relevant examples that the information premium can for example explain a sign shift of the risk premium. We apply the theory of enlargement of filtration, providing a tool for quantifying the information premium for given filtrations strictly bigger than the one generated by the spot price.

In practice, the forward looking information available to the market is most likely very complex, and the concrete situations considered here is not able to fully account for this. Future investigations will focus on more sophisticated models of information flow relevant to electricity and other markets, which better explain various market conditions. Furthermore, a study of the effect of a settlement period rather than a fixed maturity time in combination with enlarged filtrations and risk-neutral probabilities will be analyzed. Finally, more empirical studies of the electricity markets to reveal the stylized facts of risk premia are called for.

Appendix A. Initial enlargements of filtrations

The following theorem due to Yaozhung Hu is formulated in [22] (the proof is available on request). It describes explictly the nature of a Brownian motion with respect to an initial enlargement of filtration (see also Jeulin [25])

Theorem A.1. *Let $U(t)$ be a standard Brownian motion for $t \in [0,T]$, and let G be a centered (mean zero) Gaussian random variable. Assume that*

$$\mathbb{E}[U(t)G] = \rho(t), \quad \mathbb{E}[G^2] = \tau \,.$$

Define the filtration

$$\mathcal{H}_t = \sigma(G, U(s); 0 \leq s \leq t).$$

Assume that $\rho(t)$ is twice continuously differentiable. Define

$$a(s) = \frac{\rho'(s)}{\tau - \int_0^s (\rho'(u))^2\, du},\ 0 \leq s \leq T\,,$$

and

$$b_t(s) = \rho''(s) \int_s^t \frac{\rho'(v)}{\tau - \int_0^v (\rho'(u))^2\, du}\, dv,\ 0 \leq s \leq t \leq T\,.$$

If $a(s)$ and $b_t(s)$ are integrable with respect to s for all $t \leq T$, then

$$\tilde{U}(t) := U(t) - \int_0^t b_t(s) U(s)\, ds - \int_0^t a(s)(G - \rho'(s)U(s))\, ds\,,$$

is a $\mathcal{H}_t$-Brownian motion.

Thm A.1 can easily be extended to a situation where the filtration is contained in $\mathcal{G}_t$. The proof is analogous to Proposition 18 in Di Nunno et al. [16].

Proposition A.2. *Let $U(t)$ be a standard Brownian motion with $\mathcal{F}_t$ being its filtration. Further, let G, $\mathcal{H}_t$, $a(s)$ and $b_t(s)$ be as in Thm A.1, and suppose that $\mathcal{G}_t$ is a filtration such that $\mathcal{F}_t \subseteq \mathcal{G}_t \subseteq \mathcal{H}_t$ for all $t \leq T$. Then*

$$\begin{aligned} \widetilde{U}(t) &\triangleq U(t) - \int_0^t b_t(s) U(s)\, ds - \int_0^t a(s) \left\{ \mathbb{E}[G \,|\, \mathcal{G}_s] - \rho'(s) U(s) \right\}\, ds \\ &= U(t) - \int_0^t (b_t(s) - a(s)\rho'(s)) U(s)\, ds - \int_0^t a(s) \mathbb{E}[G \,|\, \mathcal{G}_s]\, ds\,, \end{aligned}$$

is a $\mathcal{G}_t$-Brownian motion.

The following Proposition will help us to calculate information premia in certain situations.

Proposition A.3. *Let $t_0 \leq T \leq T_1$ be given time points, and f and g deterministic continuous functions on $[0, T_1]$. In the setting of Proposition A.2, assume the information drift is of the form*

$$\theta(t) = g(t) \mathbb{E}\left[\int_t^{T_1} f(u) dB(u) \,|\, \mathcal{G}_t \right]\,,$$

that is, $B(t) - \int_0^t \theta(s)\, ds$ is a $\mathcal{G}_t$-Brownian motion. Then

$$\mathbb{E}\left[\int_s^{T_1} f(u)\, dB(u) \,|\, \mathcal{G}_t \right] = \mathbb{E}\left[\int_t^{T_1} f(u)\, dB(u) \,|\, \mathcal{G}_t \right] e^{-\int_t^s f(u) g(u)\, du}\,, \qquad t \leq s \leq T\,.$$

Proof. We have

$$\begin{aligned} \mathbb{E}\left[\int_s^{T_1} f(u)\, dB(u) \,|\, \mathcal{G}_t \right] - \mathbb{E}\left[\int_t^{T_1} f(u)\, dB(u) \,|\, \mathcal{G}_t \right] &= -\mathbb{E}\left[\int_t^s f(u)\, dB(u) \,|\, \mathcal{G}_t \right] \\ &= -\mathbb{E}\left[\int_t^s f(u) g(u) \mathbb{E}\left[\int_u^{T_1} f(v)\, dB(v) \,|\, \mathcal{G}_u \right] du \,|\, \mathcal{G}_t \right] \end{aligned}$$

$$= -\int_t^s f(u)g(u)\mathbb{E}\left[\int_u^{T_1} f(v)\,dB(v)\,|\,\mathcal{G}_t\right]\,du\,.$$

Thus $Y(s) \triangleq \mathbb{E}\left[\int_s^{T_1} f(u)\,dB(u)\,|\,\mathcal{G}_t\right]$ fulfills the integral equation

$$Y(s) = Y(t) - \int_t^s f(u)g(u)Y(u)\,du\,,$$

whose solution is given by

$$Y(s) = Y(t)\mathrm{e}^{-\int_t^s f(u)g(u)\,du}\,,\qquad t \le s \le T\,.$$

This proves the Proposition. □

Remark A.4. *Note that we can substitute Brownian motion $B(t)$ in Prop. A.3 with any Lévy process $L(t)$ whenever $L(t) - \int_0^t \theta(s)\,ds$ is a $\mathcal{G}_t$-martingale.*

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